

Two-loop effective potential for the Wess-Zumino model in 2+1 dimensions

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Abstract

By using superfield techniques, the effective potential of the $\mathcal{N} = 1$ Wess-Zumino model in 2+1 dimensions is computed off-shell up to two loops. It is shown that supersymmetry is not broken, and that dynamical generation of mass does not occur perturbatively. We also investigate the renormalization of the effective potential and determine the renormalization group gamma and beta functions, showing that this model is infrared free. Comparison with some other results in the literature is provided.

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I. INTRODUCTION

Although supersymmetry (SUSY) is a key concept in the physics of elementary particles and fields, it is not supported (up to now) by experimental evidences. So, any realistic model involving SUSY must include some mechanism of breakdown. Many different mechanisms of breakdown have been considered in the literature. For instance, the Minimal Supersymmetric Standard Model (MSSM) with explicit soft supersymmetry breaking operators has been suggested as a way of solving the scale of grand unification and the hierarchy problems [1]. The breakdown due to instanton solutions [2], its connection with R -symmetry breaking [3] and with the Witten index [4, 5] have also been intensely investigated along the years. Yet, several variations or extensions of the models of O’Raifeartaigh and of Fayet-Iliopoulos [6], which present spontaneous SUSY breaking, have been considered and more recently, theories which exhibit metastable vacua with broken SUSY [7] have also been proposed. Another interesting question is whether a purely perturbative mechanism, i.e., a dynamical symmetry breaking induced by radiative corrections can be achieved (in this case, a mass scale would be dynamically generated).

In $3+1$ spacetime dimensions this possibility is ruled out by nonrenormalization theorems [8]. On the other hand, in $2+1$ dimensions such restriction (at least for $\mathcal{N} = 1$ SUSY) does not exist [9, 10]. The usual way of investigating the vacuum structure in quantum field theory involves the calculation of the effective potential [11]. Recently the two-loop effective potential for the three-dimensional $\mathcal{N} = 2$ Wess–Zumino (WZ) model was evaluated in Ref. [12]. For the case $\mathcal{N} = 1$, the effective potential for the WZ model and massless electrodynamics up to one-loop were first calculated in [13] long ago. In both models, that author showed that neither SUSY nor the gauge invariance are broken by radiative corrections up to one loop order. Nevertheless, in $2+1$ dimensions, terms involving logarithms of the classical fields only appear in two or more loops. Since these logarithmic contributions have a crucial role in the dynamical symmetry breakdown, the calculations must be carried up at least to two loops.

In the component field formalism the two-loop effective potential of the WZ model was evaluated off-shell and on-shell in Refs. [14] and [15], respectively. In Ref. [14], it is reported a problem with the renormalization of the effective potential: a divergent term which can not be absorbed by rescaling of the classical Lagrangian appears. On the other hand, in

Ref. [15], difficulties with the renormalization are not found, but it is claimed that SUSY is broken and a dynamical mass generation takes place. In that paper, however, the evaluation of the effective potential did not take into account radiative corrections to the equation of motion of the auxiliary field [18]. These facts lead us to conclude that the renormalization and the vacuum structure to the three-dimensional WZ model are issues not yet satisfactorily answered.

The present work aims to calculate the two-loop effective potential of the WZ model by using the superfield formulation. We claim that the renormalization of the effective potential with dimensional reduction regularization is achieved in the usual way. Moreover, we show that SUSY is not broken and dynamical generation of mass is not perturbatively consistent. We have also determined the beta function associated with the fourfold self-interaction and verified that it agrees with the result in Ref. [19], which is obtained by direct calculation of the one-particle irreducible (1PI) Green's functions in components fields. The anomalous dimension of the superfield is also determined.

The paper is organized as follows. In Sec. II, the model is defined and the tree level potential is analyzed for different set-ups of the coupling constants. In Sec. III, the effective potential in one and two-loop order is calculated and its renormalization is analyzed, for the most general WZ model of a single real scalar superfield. In Sec. IV the possibility of dynamical symmetry breakdown, for the (sub) model that is classically scale invariant, is studied with the conclusion that the symmetries are preserved. The beta function of the coupling constant is also calculated, showing that the model has a Landau pole in the UV limit. In Sec. V, we summarize our conclusions. In Appendix A the ζ -function method for the calculation of the one-loop contribution is outlined and in Appendix B some details of the two-loops calculations are presented.

II. THE MODEL

The most general renormalizable action for the $\mathcal{N} = 1$ WZ model, containing a single real scalar superfield in $2 + 1$ dimensions is given by

$$S[\Phi] = \int d^5z \left\{ -\frac{1}{4} D^\alpha \Phi D_\alpha \Phi + \frac{1}{2} m \Phi^2 + \frac{\lambda}{3!} \Phi^3 + \frac{g}{4!} \Phi^4 + a \Phi + \mathcal{L}_{CT} \right\}, \quad (1)$$

where $d^5z \equiv d^3x d^2\theta$, $\Phi(x, \theta) = \phi(x) + \theta^\alpha \psi_\alpha(x) - F(x)\theta^2$ and \mathcal{L}_{CT} is the counterterm Lagrangian. Our conventions and notations for the superfield formalism are the same as in Ref. [20]. The mass dimensions of the scalar superfield and the coupling constants are: $[\Phi] = 1/2$, $[\lambda] = 1/2$, $[g] = 0$, $[a] = 3/2$. When $\lambda = a = 0$, the classical action is invariant under the discrete symmetry transformation $\Phi \rightarrow -\Phi$ and if in addition $m = 0$ the model is also classically scale invariant.

The component form of Eq. (1) is easily obtained by doing the θ -integration:

$$S = \int d^3x \left\{ \frac{1}{2}(\phi \square \phi + \psi^\alpha i \partial_\alpha^\beta \psi_\beta + F^2) + m(\psi^2 + \phi F) \right. \\ \left. + \lambda(\phi \psi^2 + \frac{1}{2}\phi^2 F) + \frac{g}{6}\phi^3 F + \frac{g}{2}\phi^2 \psi^2 + aF + \mathcal{L}_{CT} \right\}. \quad (2)$$

The above action is invariant under the SUSY transformations

$$\begin{aligned} \delta\phi &= -\epsilon^\alpha \psi_\alpha, \\ \delta\psi_\alpha &= -\epsilon^\beta (C_{\alpha\beta} F + i \partial_{\alpha\beta} \phi), \\ \delta F &= -\epsilon^\alpha i \partial_\alpha^\beta \psi_\beta, \end{aligned} \quad (3)$$

where ϵ^α is a constant fermionic parameter.

The tree level off-shell scalar potential is read off directly from Eq. (2):

$$V^{(0)}(\phi, F) = -\frac{1}{2}F^2 - F(a + m\phi + \frac{\lambda}{2}\phi^2 + \frac{g}{6}\phi^3). \quad (4)$$

By eliminating the auxiliary field F through its equation of motion $\partial V^{(0)}/\partial F = 0$, the classical potential becomes only a function of the physical field ϕ , such that

$$V^{(0)}(\phi) = \frac{1}{2}(F(\phi))^2 \geq 0, \quad (5)$$

with $F(\phi) = -(a + m\phi + \frac{\lambda}{2}\phi^2 + \frac{g}{6}\phi^3)$.

As is well known, for any unbroken supersymmetric theory the vacuum state must corresponds to a global minimum of the effective potential with $F(\phi_{min}) = 0$ and $V(\phi_{min}) = 0$ [21]. For $g \neq 0$ the model (at tree level) has a SUSY preserving phase, since $F(\phi_{min}) = 0$ always has at least one real solution for ϕ_{min} . In this case, if $a = \lambda = 0$ we have a minimum at $\phi_{min} = 0$ and if besides that, we also have $-6m/g > 0$, there exist two other solutions: $\phi_{min} = \pm\sqrt{-6m/g}$ that spontaneously break the symmetry $\phi \rightarrow -\phi$. Anyway, for $g \neq 0$ SUSY is classically preserved. Another possibility is when $g = 0$ and $\lambda \neq 0$. If $2a\lambda \leq m^2$, the equation $F = 0$ has two real solutions $\phi_{min} = -\frac{m}{\lambda} \pm (\frac{m^2}{\lambda^2} - \frac{2a}{\lambda})^{1/2}$ and SUSY is preserved. If instead, $2a\lambda > m^2$, the minimum of

$V^{(0)}(\phi)$ occurs for $\phi_{min} = -\frac{m}{\lambda}$ (solution of $dV^{(0)}/d\phi = 0$, for which $F = \frac{m^2}{2\lambda} - a \neq 0$) and implies in $V(\phi_{min}) = \frac{1}{8\lambda^2}(2\lambda a - m^2)^2 > 0$, showing a spontaneous breakdown of SUSY at classical level.

III. THE EFFECTIVE POTENTIAL

There are several methods by which one can calculate the effective potential in ordinary field theory. We will employ the Jackiw's functional method [22] whose extension to superspace is straightforward. The recipe is: shift the quantum superfield Φ by a classical superfield ϕ_{cl} and consider the action

$$\hat{S}[\Phi, \phi_{cl}] \equiv S[\Phi + \phi_{cl}] - S[\phi_{cl}] - \int d^5 z \Phi \left. \frac{\delta S}{\delta \Phi} \right|_{\Phi=\phi_{cl}}, \quad (6)$$

where $\phi_{cl}(\theta) = \sigma_1 - \theta^2 \sigma_2$, with $\sigma_1 = \langle \phi \rangle$ and $\sigma_2 = \langle F \rangle$ being the constant vacuum expectation values of the scalar component fields (the Lorentz invariance of the vacuum requires that $\langle \psi^\alpha \rangle = 0$).

The action \hat{S} takes the form

$$\begin{aligned} \hat{S}[\Phi, \phi_{cl}] = & \int d^5 z \left[\frac{1}{2} \Phi \left(D^2 + m + \lambda \phi_{cl} + \frac{g}{2} \phi_{cl}^2 \right) \Phi \right. \\ & \left. + \frac{1}{3!} (\lambda + g \phi_{cl}) \Phi^3 + \frac{g}{4!} \Phi^4 \right]. \end{aligned} \quad (7)$$

The effective potential can be written in a manifestly supercovariant form as

$$V_{eff}(\sigma_1, \sigma_2) = V^{(0)}(\sigma_1, \sigma_2) - \frac{i}{2\Omega} \ln \text{Det} [i\Delta_F^{-1}(z, z')] + \frac{i}{\Omega} \left\langle 0 \left| T \exp i \int d^5 z \hat{\mathcal{L}}_{int}(\Phi, \phi_{cl}) \right| 0 \right\rangle. \quad (8)$$

The first term in Eq. (8) is the tree-level potential as given in (4). The second term is the one-loop correction, where

$$i\Delta_F^{-1}(z, z') = \left. \frac{\delta^2 S[\Phi]}{\delta \Phi_z \delta \Phi_{z'}} \right|_{\Phi=\phi_{cl}} = \left(D^2 + m + \lambda \phi_{cl} + \frac{g}{2} \phi_{cl}^2 \right) \delta^5(z - z'), \quad (9)$$

and $\Omega \equiv \int d^3 x$ is the spacetime volume. The third term encodes the higher loops corrections: the sum of one-particle-irreducible vacuum superdiagrams with two and more loops computed from the shifted action (7). Let us note that the effective potential is only function of the constant (x^μ independent) fields σ_1 and σ_2 . Actually, the superfield approach adopted here guarantees that after all the D -algebra manipulations, only a single θ -integration remains to be done. This allows us to read off the effective potential as it was previously made in (2) and (4).

A. One-loop contribution

The one-loop contribution $V^{(1)}$ to the effective potential is enclosed by the functional determinant in (8). It can be evaluated by the ζ -function method as described in Ref. [13]. Following the calculations outlined in Appendix (A) we get

$$\begin{aligned} V^{(1)} &= -\frac{i}{2} \int \frac{d^3 k}{(2\pi)^3} \ln \left[\frac{k^2 + M^2}{k^2 + \mu_1^2} \right] \\ &= -\frac{1}{12\pi} (M^3 - \mu_1^3), \end{aligned} \quad (10)$$

where $M^2 = \mu_1^2 - \mu_2^2$, $\mu_1 = m + \lambda\sigma_1 + \frac{g}{2}\sigma_1^2$ and $\mu_2^2 = \lambda\sigma_2 + g\sigma_1\sigma_2$ (note that μ_2^2 may assume negative values).

Therefore, up to one-loop order, the effective potential is finite and is given by

$$\begin{aligned} V_{eff}(\sigma_1, \sigma_2) &= V^{(0)} + V^{(1)} \\ &= -\frac{1}{2}\sigma_2^2 - \sigma_2(a + m\sigma_1 + \frac{\lambda}{2}\sigma_1^2 + \frac{g}{6}\sigma_1^3) \\ &\quad - \frac{1}{12\pi} \left[\left[\left(m + \lambda\sigma_1 + \frac{g}{2}\sigma_1^2 \right)^2 - \lambda\sigma_2 - g\sigma_1\sigma_2 \right]^{3/2} \right. \\ &\quad \left. - \left(m + \lambda\sigma_1 + \frac{g}{2}\sigma_1^2 \right)^3 \right]. \end{aligned} \quad (11)$$

From this expression it is easy to see that $V_{eff}(\sigma_1, \sigma_2 = 0) = 0$. So, if $\sigma_2 = 0$ corresponds to an absolute minimum of V_{eff} then SUSY is not broken. This is the case for $g > 0$. Indeed, the constraint

$$\left. \frac{\partial V_{eff}}{\partial \sigma_1} \right|_{\sigma_2=0} = 0, \quad (12)$$

is identically satisfied and the equation

$$\left. \frac{\partial V_{eff}}{\partial \sigma_2} \right|_{\sigma_2=0} = 0, \quad (13)$$

implies in the condition

$$\left(a - \frac{m\lambda}{8\pi} \right) + \left(m - \frac{gm}{8\pi} - \frac{\lambda^2}{8\pi} \right) \sigma_1 + \left(\frac{\lambda}{2} - \frac{3g\lambda}{16\pi} \right) \sigma_1^2 + \left(\frac{g}{6} - \frac{g^2}{16\pi} \right) \sigma_1^3 = 0. \quad (14)$$

For $g \neq 0$, the above equation has at least one real solution for σ_1 , what means that SUSY is not broken, as it happens at tree level. In the particular case $a = \lambda = m = 0$, we have $\sigma_1 = \sigma_2 = 0$ as an absolute minimum and the discrete symmetry and the scale symmetry are both preserved. Our calculations agree with the result in Ref. [13].

If instead $g = 0$ and $\lambda \neq 0$, we have real solutions for σ_1 if $2a\lambda \leq m^2 + \frac{\lambda^4}{64\pi^2}$, meaning that SUSY is realized in one loop order. Remembering that SUSY was broken at classical level when

$g = 0$ and $2a\lambda > m^2$, we see that a restoration of SUSY will occur due to one-loop corrections if $m^2 < 2a\lambda < m^2 + \frac{\lambda^4}{64\pi^2}$.

B. Two-loop contribution

As is well known, for symmetry breaking to occurs by radiative corrections, we need the induction of terms of the form $h(\sigma_1, \sigma_2) \ln f(\sigma_1, \sigma_2)$. In $2 + 1$ dimensions, this only happens in two (or more) loops approximation. To study this possibility and to make a detailed analysis of the UV counterterms needed to renormalize the effective potential, we will consider the general case in which all the parameters in Eq. (1) are non-null.

Let us start by establishing the supergraph Feynman rules for the shifted theory (7). The Feynman propagator satisfies the Green equation:

$$\hat{\mathcal{O}}_z \Delta_F(z - z') = i\delta^5(z - z'), \quad (15)$$

where $\hat{\mathcal{O}}_z = D_z^2 + \mu_1 - \mu_2^2 \theta^2$ with μ_1 and μ_2^2 defined as before.

To invert the operator $\hat{\mathcal{O}}$ we make use of the projection operators method, developed in Ref. [23]. A basis for the space of scalar operators is formed by the set of six linearly independent operators:

$$P_0 = 1, \quad P_1 = D^2, \quad P_2 = \theta^2, \quad P_3 = \theta^\alpha D_\alpha, \quad P_4 = \theta^2 D^2, \quad P_5 = i\partial_{\alpha\beta} \theta^\alpha D^\beta,$$

with the following multiplication table:

	P_1	P_2	P_3	P_4	P_5
P_1	\square	$-P_0 + P_3 + P_4$	$2P_1 + P_5$	$-P_1 + \square P_2 - P_5$	$\square(-2P_0 + P_3)$
P_2	P_4	0	0	0	0
P_3	$-P_5$	$2P_2$	$P_3 - 2P_4$	$2P_4$	$2\square P_2 + P_5$
P_4	$\square P_2$	$-P_2$	$2P_4$	$-P_4$	$-2\square P_2$
P_5	$-\square P_3$	0	$-2\square P_2 + P_5$	0	$\square(P_3 + 2P_4)$

TABLE I: Multiplication table employed in the inversion of $\hat{\mathcal{O}}$. In addition, one have the trivial relations: $P_0 P_i = P_i P_0 = P_i$, with $i = 0, \dots, 5$.

After a straightforward algebra, the superpropagator in momentum space is given by

$$\Delta_F(k; \theta - \theta') = i \left(\sum_{i=0}^5 c_i P_i \right) \delta^2(\theta - \theta'), \quad (16)$$

where:

$$\begin{aligned} c_0 &= \frac{\mu_1}{k^2 + M^2}, \quad c_1 = -\frac{1}{k^2 + M^2}, \quad c_2 = -\frac{(k^2 - \mu_1^2)\mu_2^2}{(k^2 + \mu_1^2)(k^2 + M^2)}, \quad c_3 = -\frac{\mu_1\mu_2^2}{(k^2 + \mu_1^2)(k^2 + M^2)}, \\ c_4 &= -\frac{2\mu_1\mu_2^2}{(k^2 + \mu_1^2)(k^2 + M^2)}, \quad c_5 = -\frac{\mu_2^2}{(k^2 + \mu_1^2)(k^2 + M^2)}. \end{aligned}$$

The interaction vertices may be read from Eq. (7) and the symmetry factors can be determined by the Wick's theorem in the conventional way.

The two-loop superdiagrams contributing for the effective potential are drawn in Fig. 1. The associated analytical expressions are shown in Appendix (B) and the resulting two-loop momentum integrals are evaluated by dimensional regularization using the formulas presented in Ref. [24]. The contribution of the diagram (a), denoted by $V_a^{(2)}$, results to be finite, since it is constituted by the product of nonoverlapping one-loop integrals. Diagram (b), instead has divergences proportional to all the terms present in the tree-level potential $V^{(0)}$, which is consistent with the usual renormalizability of the model. In summary, we have the following results:

$$\begin{aligned} V_a^{(2)} &= -\frac{g}{32\pi^2} \frac{M\mu_1\mu_2^2}{(M + \mu_1)}, \\ V_b^{(2)} &= \frac{(\lambda + g\sigma_1)^2}{64\pi^2} \left[\frac{\mu_2^2}{2} I_{div} - 6\mu_1^2 \ln \left(\frac{2M + \mu_1}{\mu} \right) + (M^2 + 5\mu_1^2) \ln \left(\frac{3M}{\mu} \right) \right] \\ &\quad + \frac{(\lambda + g\sigma_1)^2}{64\pi^2} \left[-M^2 \ln \left(\frac{M}{\mu} \right) + \frac{M^2}{3} \left\{ 1 + \ln \left(\frac{M + 2\mu_1}{27\mu} \right) \right\} - \frac{2}{3} M\mu_1 \right. \\ &\quad \left. + \frac{\mu_1^2}{3} \left\{ 1 - 6 \ln \left(\frac{3M}{\mu} \right) - 10 \ln \left(\frac{M + 2\mu_1}{\mu} \right) \right\} + \frac{2}{3} (M^2 + 8\mu_1^2) \ln \left(\frac{2M + \mu_1}{\mu} \right) \right] \\ &\quad + \frac{g\sigma_2}{64\pi^2} \left[\left\{ I_{div} - 2 \ln \left(\frac{3M}{\mu} \right) \right\} \left(\lambda\mu_1 + g\mu_1\sigma_1 - \frac{g}{6}\sigma_2 \right) \right], \end{aligned} \quad (17)$$

where $I_{div} = \frac{1}{\epsilon} + \ln[4\pi e^{(1-\gamma_E)}]$ and μ is an arbitrary mass parameter introduced via dimensional regularization.

The effective potential up to two loops is given by

$$V_{eff} = V^{(0)} + V^{(1)} + V_a^{(2)} + V_b^{(2)} + V_{CT}, \quad (18)$$

in which V_{CT} is the counterterm contribution to the potential

$$V_{CT} = - \left[\frac{1}{2} \delta Z \sigma_2^2 + \delta m \sigma_1 \sigma_2 + \frac{\delta \lambda}{2} \sigma_1^2 \sigma_2 + \frac{\delta g}{6} \sigma_1^3 \sigma_2 + \delta a \sigma_2 \right], \quad (19)$$

as can be read from the classical Lagrangian in Eq.(4); δZ is the wave function renormalization counterterm and the other counterterms are self explaining. The divergent parts of $V^{(2)}$ can be collected in

$$V_{div}^{(2)} = \frac{I_{div}}{128\pi^2} \left[-\frac{1}{3}g^2\sigma_2^2 + (2g^2m + 5g\lambda^2)\sigma_1\sigma_2 + 6g^2\lambda\sigma_1^2\sigma_2 + 2g^3\sigma_1^3\sigma_2 + (2gm\lambda + \lambda^3)\sigma_2 \right] \quad (20)$$

As seen from this equation, the renormalization of the effective potential requires all the counterterms in Eq. (19):

$$\begin{aligned} \delta Z &= -\frac{1}{3}\hat{g}^2 I_{div} + \delta Z_{fin} \\ \delta a &= \frac{1}{2}(2m\hat{g}\hat{\lambda} + \lambda\hat{\lambda}^2)I_{div} + \delta a_{fin} \\ \delta m &= \frac{1}{2}(2m\hat{g}^2 + 5g\hat{\lambda}^2)I_{div} + \delta m_{fin} \\ \delta \lambda &= 6\hat{g}^2\lambda I_{div} + \delta \lambda_{fin} \\ \delta g &= 6g\hat{g}^2 I_{div} + \delta g_{fin} \end{aligned} \quad (21)$$

where we defined $\hat{g} = g/8\pi$ and $\hat{\lambda} = \lambda/8\pi$.

Let us compare our results with some others in the literature. In [12] the effective potential of the $\mathcal{N} = 2$ WZ model in 2+1 D was studied in two loops approximation. The authors conclude that only a wave function renormalization is needed. As they say, that result is not unexpected; the $\mathcal{N} = 2$ superspace formulation of supersymmetry in 2+1 D can be got from the $\mathcal{N} = 1$ superspace formulation in 3+1 D by dimensional reduction and so, the 3+1 D nonrenormalization theorems are expected to work with $\mathcal{N} = 2$ in 2+1 D supersymmetry. In our results on the other side, no nonrenormalization theorem applies and the renormalization of all the parameters are necessary. Differently from ours, in which three different arguments appear in the generated logarithms, their expression has only a single argument in the generated logarithms. This difference is, maybe, due to their approximation, in which spinorial derivatives $D_\alpha\Phi$ and $D^2\Phi$, besides the usual spatial $\partial\Phi/\partial x^\mu$ are dropped. Our results also contradict the result for a similar $\mathcal{N} = 1$ model, reported in Ref. [14], in which a counterterm of the form σ_1^6 , not present in the classical Lagrangian, was found to be required.

For the model with $g \neq 0$ and $\lambda \neq 0$ the renormalization also requires that δa and δm be non null. The sub model with only $g \neq 0$ is renormalizable, that is, it only requires the renormalization of g besides that of Z . If $g = 0$ and $\lambda \neq 0$ (in which case the model is super-renormalizable) the cancellation of the UV divergencies in two loops requires also that $\delta a \neq 0$.

IV. THE UNBROKEN SUSY VACUUM

Let us now investigate in more details the sub model with $g \neq 0$ and $m = \lambda = a = 0$, that is of particular interest for being classically scale invariant. As discussed in the previous section, the model only requires the δZ and δg counterterms. The total renormalized effective potential V_{eff} takes the form

$$V_{eff} = -\frac{1 + \delta Z_{fin}}{2} \sigma_2^2 - \frac{g + \delta g_{fin}}{6} \sigma_1^3 \sigma_2 + \frac{1}{12\pi} (\mu_1^3 - M^3) \\ + \frac{g}{32\pi^2} \left[\frac{1}{3} \mu_1 (\mu_1 - M) (\mu_1 - 4M) - \frac{2}{3} \mu_1 (\mu_1^2 - M^2) \ln \frac{2M + \mu_1}{\mu} \right. \\ \left. - \frac{1}{3} \mu_1 (10\mu_1^2 - M^2) \ln \frac{M + 2\mu_1}{\mu} + \left(\mu_1 (2\mu_1^2 + M^2) + \frac{g}{6} \sigma_2^2 \right) \ln \frac{3M}{\mu} \right] \quad (22)$$

where $\mu_1 = g\sigma_1^2/2$, $\mu_2^2 = g\sigma_1\sigma_2$ and $M = (\mu_1^2 - \mu_2^2)^{1/2}$. Observe that V_{eff} is real only for M real, that is, if $(g\sigma_1^4 - 4\sigma_1\sigma_2) > 0$. The singularity in $\sigma_1 = 0$ for $\sigma_2 \neq 0$, in the last term of V_{eff} , is a reminiscence of the IR divergencies due to the null mass of the model. So, $\sigma_1 = 0$ is not a convenient spot to impose renormalization conditions. The point $\sigma_1^2 = \mu$, where μ is the mass parameter introduced by the dimensional regularization, is a more natural spot. To see this fact, let us expand the expression of the effective potential in powers of σ_2 . The result is

$$V_{eff} = \sigma_2 \sigma_1^3 \left[-\frac{g}{6} + \left(-\frac{\delta g_{fin}}{6} - \frac{1}{2} g\hat{g} + \frac{1}{2} g\hat{g}^2 - \frac{3}{2} g\hat{g}^2 - 2g\hat{g}^2 \ln \frac{2g}{3} \right) - 2g\hat{g}^2 \ln \frac{\sigma_1^2}{\mu} \right] \\ + \sigma_2^2 \left[-\frac{1}{2} + \left(-\frac{\delta Z_{fin}}{2} - \frac{1}{2} \hat{g} + \frac{29}{18} \hat{g}^2 - 2g\hat{g}^2 + \frac{1}{3} \hat{g}^2 \ln \frac{2g}{3} \right) + \frac{1}{3} \hat{g}^2 \ln \frac{\sigma_1^2}{\mu} \right] + \sigma_2^3 \mathcal{F}(\sigma_1), \quad (23)$$

where, as before, $\hat{g} = g/8\pi$. We choose δg_{fin} and δZ_{fin} by imposing that the terms in the parentheses be nulls. These choices imply that at the point $\sigma_1^2 = \mu$, the coefficients of the two monomials ($\sigma_2 \sigma_1^3$ and σ_2^2) are the same as in the classical potential $V_{class} = -(g/6) \sigma_2 \sigma_1^3 - (1/2) \sigma_2^2$. The first condition fix the renormalized coupling constant and the second implies that the coefficient of the kinetic term of the effective renormalized Lagrangian at $\sigma_1 = \mu$ is one. In the expanded form, the renormalized potential results in

$$V_{eff} = \sigma_2 \sigma_1^3 \left(-\frac{g}{6} - 2g\hat{g}^2 \ln \frac{\sigma_1^2}{\mu} \right) \\ + \sigma_2^2 \left(-\frac{1}{2} + \frac{1}{3} \hat{g}^2 \ln \frac{\sigma_1^2}{\mu} \right) + \sigma_2^3 \mathcal{F}(\sigma_1) \quad (24)$$

Let us now investigate the possibility of supersymmetry breakdown. It is easy to check that $V_{eff}(\sigma_1, \sigma_2 = 0) = 0$, from which it also follows that $\partial V_{eff}/\partial \sigma_1|_{\sigma_2=0} \equiv 0$. The condition $\partial V_{eff}/\partial \sigma_2|_{\sigma_2=0} = 0$, leads to the following (gap) equation for σ_1 :

$$\sigma_1^3 \left(-\frac{g}{6} - 2g\hat{g}^2 \ln \frac{\sigma_1^2}{\mu} \right) = 0 \quad (25)$$

This equation has a trivial solution $\sigma_1^{min} = 0$ that ensures that SUSY as well as the discrete symmetry $\Phi \rightarrow -\Phi$ are not broken by the radiative corrections. Looking at the term in the parentheses, a possible non-zero solution $\sigma_1^{min} \neq 0$ would be given by

$$g^2 \ln \frac{(\sigma_1^{min})^2}{\mu} = -5\pi^2. \quad (26)$$

Following Coleman-Weinberg's arguments [11]: since higher loop orders are expected to bring in higher powers of $g^2 \ln \frac{\sigma_1^2}{\mu}$ and so, at this value of $(\sigma_1^{min})^2$, higher powers of $5\pi^2 \gg 1$, this minimum lies very far from the range of validity of the two loop approximation; we conclude that, no non-trivial vacuum is induced by radiative corrections and no SUSY breaking and mass generation occur. Our results contradict the claim made in Ref. [15], that the two-loop corrections are able to induce supersymmetry breaking and dynamic generation of mass. On the other hand a similar conclusion to ours was obtained in [16], for the O(N) WZ model in $1/N$ approximation. The same conclusion was also got in [17] through a functional renormalization group analyses. In fact, as discussed in [11], spontaneous symmetry breaking and mass generation through radiative corrections are only possible in models with more than one coupling constant and is made possible through an interplay among these constants.

Finally, let us determine the renormalization group function β_g for the particular case with $g \neq 0$ and $m = \lambda = a = 0$. Introducing the bare Φ_0 and renormalized superfield Φ and the renormalized coupling constant g through the definitions

$$\Phi_0 = Z_\Phi^{\frac{1}{2}} \Phi = (1 + \delta Z)^{\frac{1}{2}} \Phi, \quad (27)$$

$$g_0 = \mu^\varepsilon g Z_g = \mu^\varepsilon g \left[\frac{1 + g^{-1} \delta g}{Z_\Phi^2} \right], \quad (28)$$

and writing explicitly the counterterms from Eq. (21) as

$$\delta Z = -\frac{g^2}{192\pi^2} \frac{1}{\varepsilon} + \text{finite}, \quad (29)$$

$$\delta g = \frac{3g^3}{32\pi^2} \frac{1}{\varepsilon} + \text{finite}, \quad (30)$$

we obtain the beta function at leading order

$$\begin{aligned} \beta_g &= \mu \frac{\partial g}{\partial \mu} = \frac{5g^3}{24\pi^2} - \varepsilon g \\ &= \frac{5g^3}{24\pi^2} \quad (\text{for } \varepsilon \rightarrow 0). \end{aligned} \quad (31)$$

This result is in agreement with that obtained in Ref. [19] by calculating the divergent parts of several vertex functions in the component field formalism. The solution of Eq. (31) is given by

$$\bar{g}^2 = \frac{g^2}{1 - \frac{5}{12\pi^2} g^2 \ln \frac{\bar{\mu}}{\mu}}. \quad (32)$$

Starting with a $g^2 \ll 1$ at a scale μ , we see that the effective coupling constant \bar{g}^2 increases as the scale $\bar{\mu}$ is increased. So, at short distances the above results are not reliable: higher loop corrections become more and more important compared to the second order. If instead, we make $\bar{\mu} \rightarrow 0$ we get $\bar{g}^2 \rightarrow 0$, showing an IR free limit.

An anomalous scaling of the model is also induced as can be seen by calculating the anomalous dimension of the field:

$$\gamma_\Phi = \frac{1}{2}\mu \frac{d \ln Z_\Phi}{d\mu}. \quad (33)$$

From (27), we can write (33) in the form

$$2(1 + \delta Z) \gamma_\Phi = \mu \frac{\partial \delta Z}{\partial g} \frac{\partial g}{\partial \mu}. \quad (34)$$

By replacing (29) and (31) into (34) we get

$$2 \left(1 - \frac{g^2}{192\pi^2} \frac{1}{\varepsilon} \right) \gamma_\Phi = \frac{g^2}{96\pi^2} - \frac{5g^4}{24 \cdot 96\pi^4} \frac{1}{\varepsilon}, \quad (35)$$

which yields

$$\gamma_\Phi = \frac{g^2}{192\pi^2}. \quad (36)$$

V. CONCLUSIONS

In the present paper, we calculate the effective potential for the $\mathcal{N} = 1$ WZ model in $2 + 1$ dimensions. We employ the Jackiw's functional method combined with the superfields formalism. A detailed analysis of the renormalizability and vacuum structure of the model is presented, up to two loops. One of the main results is that the renormalization of the theory requires, besides the wave function counterterm, also mass and coupling constants counterterms, but not any new one. This result differs from that reported in [14], where the renormalization of the model requires an extra σ_1^6 counterterm. It also differs from that in Ref. [12] for the $\mathcal{N} = 2$ WZ model in $2 + 1$ D in which only a wave function renormalization was found to be required. For the massless Φ^4 (sub) model, we also determined the β_g function which agrees with the results of [19], showing a Landau pole in UV limit. At the same time, it was found that the quantum vacuum state preserves supersymmetry and the discrete symmetry $\Phi \rightarrow -\Phi$ of the classical theory, contrary to the remark in [15], but in agreement with the results in [16] and [17].

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Appendix A: The ζ -function method

In this appendix we compute the one-loop contribution $V^{(1)}$ by the ζ -function method following Ref. [13]. The functional determinant $Det\hat{\mathcal{O}}$ is understood as the product of the eigenvalues of $\hat{\mathcal{O}}$. Starting with the eigenvalues equation

$$\int d^5 z' \hat{\mathcal{O}}_z(z, z') f_n(z') = \alpha_n f_n(z), \quad (\text{A1})$$

and defining the ζ -function associated to $\hat{\mathcal{O}}(z, z') \equiv \hat{\mathcal{O}}_z \delta^5(z - z')$ as

$$\zeta(s) = \sum_n \frac{1}{\alpha_n^s}, \quad (\text{A2})$$

the functional determinant of $\hat{\mathcal{O}}_z$ can be written in the form

$$Det\hat{\mathcal{O}}_z \equiv \prod_n \alpha_n = \exp[-\zeta'(0)]. \quad (\text{A3})$$

So, the calculation of the determinant requires to get an analytical representation for $\zeta(s)$. To this end, let us introduce a two-point superspace function $G(z, z'; \tau)$ which obeys the equation:

$$\hat{\mathcal{O}}_z G(z, z'; \tau) + \frac{\partial G}{\partial \tau} = 0, \quad (\text{A4})$$

with the initial condition $G(x, \theta; x', \theta'; \tau = 0) = \delta^3(x - x') \delta^2(\theta - \theta')$.

It is straightforward to check that

$$\zeta(s) = \frac{1}{\Gamma(s)} \int_0^\infty d\tau \tau^{s-1} \int d^3 x d^2 \theta G(x = x', \theta = \theta'; \tau), \quad (\text{A5})$$

for $G(z, z'; \tau) \equiv \sum_n \exp[-\alpha_n \tau] f_n(z) f_n^*(z')$.

To proceed we must now determine an explicit solution of $G(z, z'; \tau)$ satisfying the Eq. (A4) subject to the initial condition above. To this aim, we will assume that this function is spacetime translational invariant so that it can be written as:

$$G(x, \theta; x', \theta'; \tau) = \int \frac{d^3 k}{(2\pi)^3} g(k, \theta, \theta'; \tau) \exp[-ik(x - x')], \quad (\text{A6})$$

with the following ansatz for $g(k, \theta, \theta'; \tau)$:

$$g(k, \theta, \theta'; \tau) = A(k, \tau) + \theta^\alpha \theta'^\beta k_{\alpha\beta} B(k, \tau) + \theta^\alpha \theta'_\alpha C(k, \tau) + \theta^2 D(k, \tau) + \theta'^2 E(k, \tau) + \theta^2 \theta'^2 H(k, \tau). \quad (\text{A7})$$

To find the coefficients A , B , C , D , E and H , we have to use the explicit form of \hat{O}_z read off from (15) and insert (A7) into (A4). This equation splits into six linear ordinary differential equations with the initial conditions:

$$\begin{aligned} A(k, 0) &= 0 & B(k, 0) &= 0 & C(k, 0) &= 1 \\ D(k, 0) &= -1 & E(k, 0) &= -1 & H(k, 0) &= 0, \end{aligned} \quad (\text{A8})$$

so that the solution of this system is readily found. From these results, we now construct the ζ -function as prescribed in Eq. (7). After integration and using the relation $V^{(1)} = -(i/2\Omega) \ln \text{Det} \hat{O} = (i/2\Omega) \zeta'(0)$, we are able to get the result described in (10).

Appendix B: Two-loop diagrams

The analytical expressions for the two-loop vacuum bubbles that contribute to the effective potential displayed in Fig. 1 are ($d^D k \equiv \mu^\varepsilon d^{3-\varepsilon} k$):

$$\begin{aligned} V_a^{(2)} &= -\frac{g}{8} \int \frac{d^D k d^D q}{(2\pi)^{2D}} d^2 \theta \Delta_F(k; \theta - \theta_1)|_{\theta=\theta_1} \Delta_F(q; \theta - \theta_2)|_{\theta=\theta_2} \\ &= -\frac{g}{2} \int \frac{d^D k d^D q}{(2\pi)^{2D}} \left[\frac{\mu_1 \mu_2^2}{(k^2 + M^2)(q^2 + \mu_1^2)(q^2 + M^2)} \right], \end{aligned} \quad (\text{B1})$$

and

$$V_b^{(2)} = -3i \int \frac{d^D k d^D q}{(2\pi)^{2D}} d^2 \theta_1 d^2 \theta_2 \mathcal{I}(\theta_1^2, \theta_2^2) \Delta_F(k; \theta_1 - \theta_2) \Delta_F(q; \theta_1 - \theta_2) \Delta_F(-k - q; \theta_1 - \theta_2), \quad (\text{B2})$$

where

$$\mathcal{I}(\theta_1^2, \theta_2^2) = \frac{1}{36} [(\lambda + g\sigma_1)^2 - (\lambda g\sigma_2 + g^2 \sigma_1 \sigma_2)(\theta_1^2 + \theta_2^2) + g^2 \sigma_2^2 \theta_1^2 \theta_2^2]. \quad (\text{B3})$$

After performing the D -algebra and carrying out the remaining θ -integration, we obtain the following two-loop momentum integrals

$$\begin{aligned} V_b^{(2)} &= \int \frac{d^D k d^D q}{(2\pi)^{2D}} \frac{-\mu_2^2 (\lambda + g\sigma_1)^2}{12(k^2 + M^2)(q^2 + M^2)(k^2 + \mu_1^2)(q^2 + \mu_1^2)[(k+q)^2 + M^2][(k+q)^2 + \mu_1^2]} \\ &\times \{ k^4(q^2 + \mu_1^2) + 2k \cdot q [(k^2 + \mu_1^2)(q^2 + \mu_1^2) - (k^2 + q^2 - (k+q)^2 + \mu_1^2)\mu_2^2] \\ &+ \mu_1^2 [q^4 - 15\mu_1^4 - 4q^2 [(k+q)^2 + 2\mu_1^2] + 6\mu_1^2 \mu_2^2 + 2(k+q)^2 (-5\mu_1^2 + \mu_2^2)] \\ &+ k^2 q^4 - 4k^2 \mu_1^2 [(k+q)^2 + 2\mu_1^2] + k^2 q^2 [2(k+q)^2 - \mu_1^2 - 4\mu_2^2] \} \\ &+ \int \frac{d^D k d^D q}{(2\pi)^{2D}} \frac{-6g\mu_1(\lambda + g\sigma_1)\sigma_2 + g^2 \sigma_2^2}{12(k^2 + M^2)(q^2 + M^2)[(k+q)^2 + M^2]}. \end{aligned} \quad (\text{B4})$$

The two-loop integrals were performed by dimensional reduction scheme, using the formulas from [24]. The final results are written in (17).

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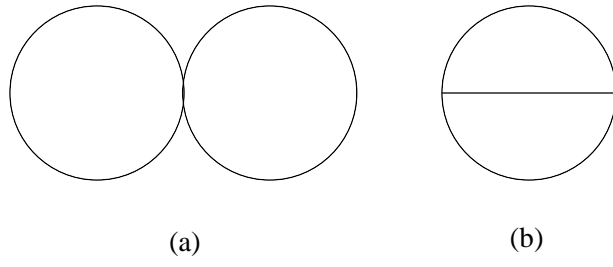


FIG. 1: Two-loop vacuum bubble supergraphs.